

Cambridge Monographs on  
Mechanics and Applied Mathematics

# **The Structure of Turbulent Shear Flow**

**Second Edition**

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Cambridge University Press

# THE STRUCTURE OF TURBULENT SHEAR FLOW

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SECOND EDITION

333/4072 INSTITUT  
FÜR METEOROLOGIE U. KLIMATOLOGIE  
UNIVERSITÄT HANNOVER  
HERKENTHAUSER STR. 2 · 30419 HANNOVER



CAMBRIDGE UNIVERSITY PRESS

CAMBRIDGE

LONDON · NEW YORK · MELBOURNE

## CONTENTS

*Preface* *page xi*

### I THE STATISTICAL DESCRIPTION OF TURBULENT FLOW

1.1	Introduction	1
1.2	The development of a theory for turbulent flow	3
1.3	The statistical description of turbulent flow	3
1.4	Notation for turbulent flows	5
1.5	Three-dimensional correlation and spectrum functions	6
1.6	One-dimensional correlation and spectrum functions	16
1.7	Correlations and spectra with time delay	23
1.8	Homogeneity and symmetry of turbulent flows	28

### 2 THE EQUATIONS OF MOTION FOR TURBULENT FLOW

2.1	Assumption of a continuous fluid	32
2.2	The equations of fluid motion	33
2.3	Approximate forms of the equations of motion	36
2.4	Mean value equations for momentum, energy and heat	38
2.5	Energy dissipation by viscosity	41
2.6	Conductive dissipation of temperature fluctuations	43
2.7	The relation between the pressure and velocity fields	43

### 3 HOMOGENEOUS TURBULENT FLOWS

3.1	Introduction	45
3.2	Eddy interactions in homogeneous turbulence	46
3.3	Experimental approximations to homogeneous turbulence	49
3.4	Isotropic turbulence: general	51
3.5	Reynolds number similarity in isotropic turbulence	53
3.6	Self-preserving development in isotropic turbulence	59
3.7	Space-time correlations in isotropic turbulence	62

3.8	The Taylor approximation of frozen flow	64
3.9	The tendency to isotropy of homogeneous turbulence	66
3.10	Uniform distortion of homogeneous turbulence	71
3.11	Irrotational distortion of grid turbulence	74
3.12	Unidirectional, plane shearing of homogeneous turbulence	80
3.13	Local isotropy and equilibrium of small eddies	88
3.14	Measurement of spectrum and structure functions	93
3.15	Energy transfer in the inertial subrange	99
3.16	The equilibrium spectrum in the viscous subrange	100
3.17	Local isotropy in non-Newtonian fluids	103

#### 4 INHOMOGENEOUS SHEAR FLOW

4.1	Large eddies and the main turbulent motion	105
4.2	Structural similarity of the main turbulent motion	106
4.3	Nature of the main turbulent motion	118
4.4	Generation and maintenance of the main motion	120
4.5	Flow inhomogeneity and the large eddies	122
4.6	The dependence of Reynolds stress on mean velocity	124
4.7	Statistical distributions of velocity fluctuations	126

#### 5 TURBULENT FLOW IN PIPES AND CHANNELS

5.1	Introduction	130
5.2	Equations of motion for unidirectional mean flow	131
5.3	Reynolds number similarity in pipe and channel flow	133
5.4	Wall similarity in the region of constant stress	135
5.5	Flow over rough walls	140
5.6	Mean flow in the central region	145
5.7	The turbulent motion in constant-stress equilibrium layers	150
5.8	Eddy structure in equilibrium layers	156
5.9	Motion in the viscous layer next the wall	163
5.10	Fluctuations of pressure and shear stress on a wall	165
5.11	The magnitude of the Kármán constant	168
5.12	Turbulent flow and flow constants	169

5.13	Similarity flows in channels and pipes of varying widths	172
5.14	Equilibrium layers with variable stress	176
5.15	Equilibrium layers with linear distributions of stress	180
5.16	Equilibrium layers with surface transpiration	184
5.17	Equilibrium layers with variable direction of flow	186

## 6 FREE TURBULENT SHEAR FLOWS

6.1	General properties of free turbulence	188
6.2	Equations of motion: the boundary-layer approximation	188
6.3	Integral constraints on free turbulent flows	193
6.4	Self-preserving development of free turbulent flows	195
6.5	The distributions of mean velocity and Reynolds stress	201
6.6	The balance of turbulent kinetic energy	205
6.7	The bounding surface of free turbulent flows	209
6.8	Distributions of turbulent intensity and Reynolds stress	214
6.9	Flow constants for self-preserving jets and wakes	220
6.10	The flow constants of plane mixing layers	227
6.11	The entrainment of ambient fluid	230
6.12	Basic entrainment processes	232
6.13	Entrainment eddies in plane wakes	241
6.14	Mechanism of the entrainment eddies	243
6.15	Control of the entrainment rate	247
6.16	Fluctuations outside the turbulent flow: sound radiation	248
6.17	Irrotational fluctuations in the near field	251
6.18	Development of nearly self-preserving flows	252
6.19	Development of a jet in a moving stream of constant velocity	255

## 7 BOUNDARY LAYERS AND WALL JETS

7.1	Wall layers in general	259
7.2	Self-preserving development of wall layers	262
7.3	General properties of self-preserving wall layers	263
7.4	Flow parameters of self-preserving wall layers	266
7.5	Development of self-preserving wall jets	268
7.6	Development of self-preserving boundary layers	272

7.7	Boundary-layer development with zero wall stress	276
7.8	Wall layers with convergent flow	280
7.9	Almost self-preserving development	283
7.10	Layers with nearly uniform velocity in the free stream	287
7.11	Turbulent flow in self-preserving boundary layers	289
7.12	Development of boundary layers in arbitrary external conditions	294
7.13	Boundary-layer development after a sudden change of external conditions	298
7.14	Development in a region of strong adverse pressure gradient	301
7.15	Layer development after a sudden change of roughness	307
7.16	Boundary layers with three-dimensional mean flow	312
7.17	Three-dimensional flow with negligible Reynolds stresses	316
7.18	Homogeneous three-dimensional flow – the Ekman layer	318
7.19	Secondary flow in a boundary layer with a free edge	323
7.20	Lateral variations of stress in boundary layers	328
7.21	Periodic structure of flow near the viscous layer	331

## 8 TURBULENT CONVECTION OF HEAT AND PASSIVE CONTAMINANTS

8.1	Governing equations and dimensional considerations	334
8.2	Diffusion by continuous movements: effect of molecular diffusive transport	336
8.3	Eulerian description of convective flows: mean value equations and correlation functions	338
8.4	Local forms of the Richardson number	341
8.5	Spectrum functions and local similarity	342
8.6	Scattering of light by density fluctuations in a turbulent flow	348
8.7	Self-preserving development of temperature fields in forced convection flows	350
8.8	Forced convection in wall flows	352
8.9	Rates of heat transfer in forced convection	356
8.10	Convection in a constant-stress layer after an abrupt change in wall flux or temperature	361
8.11	Longitudinal diffusion in pipe flow	364

8.12	Natural convection and energy transfer	366
8.13	Buoyant plumes and thermals	366
8.14	The effect of buoyancy forces on turbulent motion	372
8.15	Horizontal wall layers with heat transport	375
8.16	Nature of turbulence in strongly stable flows	378
8.17	Transient behaviour of boundary layers with heat transfer	379
8.18	Convective turbulence	380
8.19	Heat convection between horizontal, parallel planes	381
8.20	Heat transfer in Benard convection	384
8.21	Similarity and structure of Benard convection	386
8.22	Natural convection in wall layers	390

## 9 TURBULENT FLOW WITH CURVATURE OF THE MEAN VELOCITY STREAMLINES

9.1	Mean value equations for curved flow: the analogy between the effects of flow curvature and density stratification	393
9.2	Couette flow between rotating cylinders	398
9.3	Flow with the outer cylinder stationary	400
9.4	Turbulent motion with the outer cylinder stationary	404
9.5	Flow with the outer cylinder rotating	407
	<i>References</i>	413
	<i>Index</i>	425